Midterm test for Kwantumfysica 1 - 2004-2005

Friday 11 March 2005 9:15 - 10:00
Werkcollege-zalen group 1, 2, 3

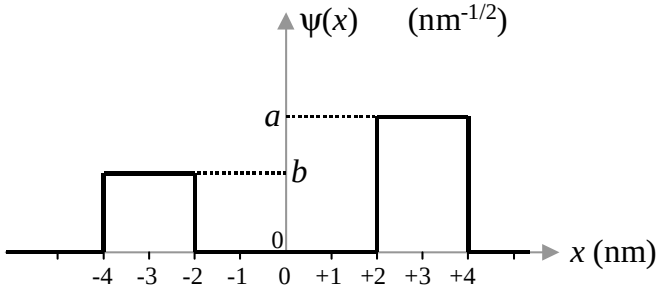
Clearly write your name and study number on each answer sheet that you use.
On the first answer sheet, write clearly the number of sheets that you turn in.

Note that this test has 2 questions, one is on the backside of this paper.

If you tend to get stuck on a problem for along time, it may be wise to skip it and try a next problem first, since this test is only 45 minutes!

Problem T1

The position x of a particle is described by the normalized, real-valued wavefunction as sketched in the figure below, with $a = \sqrt{3/10\ \text{nm}^{-1}}$ and $b = \sqrt{2/10\ \text{nm}^{-1}}$.



a) What is the expectation value $\langle \hat{x} \rangle$ for this state?

b) Show that the uncertainty Δx in the particle's position is $\sqrt{\frac{5384}{600}}$ nm .

c) With the particle's wavefunction as sketched, you plan to measure the position x. What is the probability for detecting a value in the range $\langle \hat{x} \rangle$ - 0.1 nm < x < $\langle \hat{x} \rangle$ + 0.1 nm? (If you do not have an answer for question **a)**, answer question **c)** while assuming $\langle \hat{x} \rangle$ = 0 nm, and clearly mark this on your answer form.)

d) With the particle's wavefunction as sketched, you plan to measure the position x. What is the probability for detecting a value in the range -4 nm < x < -3 nm?

Z.O.Z.

Problem T2

NOTE: if you do not find an answer for **a)**, keep C_1 as a constant in your answers for **b)-e**).

For this problem, consider a system with a time-independent Hamiltonian

$$\hat{H} = \hat{T} + \hat{V},$$

where T a kinetic-energy term and V a potential-energy term. With respect to a lowest point in the potential, defined as $V = 0$, the lowest two energy eigenstates of the system are

$$\begin{aligned} \hat{H}|\varphi_1\rangle &= E_1|\varphi_1\rangle \\ \hat{H}|\varphi_2\rangle &= E_2|\varphi_2\rangle \end{aligned},$$

where $E_1 = h\cdot(1\text{ GHz})$ and $E_2 = h\cdot(3\text{ GHz})$ the two energy eigenvalues (with h the constant of Planck), and $|\varphi_1\rangle$ and $|\varphi_2\rangle$ two orthogonal, normalized energy eigenvectors. The observable $\hat{A}$, is associated with the electric dipole A of this quantum system. For this system,

$$\begin{aligned} \langle\varphi_1|\hat{A}|\varphi_1\rangle &= A_0 \\ \langle\varphi_2|\hat{A}|\varphi_2\rangle &= 2A_0 \\ \langle\varphi_1|\hat{A}|\varphi_2\rangle &= \langle\varphi_2|\hat{A}|\varphi_1\rangle = 8A_0 \end{aligned}.$$

Note that the states $|\varphi_1\rangle$ and $|\varphi_2\rangle$ are energy eigenvectors, and that they are ***not*** eigen vectors of $\hat{A}$.

a) At some time $t = 0$, the state of the system is (with C_1 a real-valued constant)

$$|\Psi_0\rangle = \sqrt{C_1}\ |\varphi_1\rangle + \sqrt{\frac{2}{3}}\ |\varphi_2\rangle \quad .$$

For what value of C_1 is $|\Psi_0\rangle$ a normalized state?

b) What is for this state at $t = 0$ the expectation value $\langle\hat{A}\rangle$ for A, expressed in A_0? (Use C_1 as found in **a)**.

c) You measure what energy state the system is in, and find that it is in the ground state. What is the state $|\Psi_M\rangle$ of the system immediately after the measurement?

Now first read what is asked for questions **d)** and **e)**, since it follows from the same calculation:

d) Show that as a function of time $t > 0$, the expectation value for $\langle\hat{A}\rangle$ has oscillations at a frequency of 2 GHz only, for the case that the system is in $|\Psi_0\rangle$ at t=0. Use the time-evolution operator (with $\hbar = h/2\pi$)

$$\hat{U} = e^{\frac{-i\hat{H}t}{\hbar}}.$$

e) For $t > 0$, calculate the amplitude of these oscillations of the expectation value $\langle\hat{A}\rangle$ in terms of A_0 , for the case that the system was in $|\Psi_0\rangle$ at t=0.

Homework problems for this week:
5.2, 5.8, 5.12, 5.13, 5.19, 5.30, 5.33, 5.42, 5.46, 5.48, 5.50

Werkcollege problems for 2nd and 3rd werkcollege hour today:
5.1, 5.3, 5.9, 5.22, 5.28, 5.29, 5.37, 5.47, 5.52